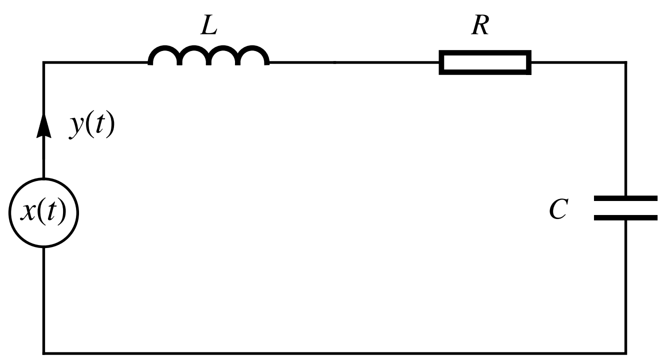
Project - Transformer, Signaler och System, 2016.

Written documentation of solutions/Matlab code including comprehensive commentary should be mailed to Ebrahim Chalangar, ebrahim.chalangar@hh.se. Oral presentation by appointment.

1. The circuit in the figure below can be described in terms of input/output of a LTIC-system, where $x(t)$ (input) is the circuit voltage and $y(t)$ (output) is the current. Assuming that all circuit components - resistor, capacitor and inductor - behave ideally, the current for $t \geq 0$ is then given by the following linear integro-differential equation (we also assume that the capacitor charge is $q = 0$ at $t = 0$ and disregard the initial condition $y(0)$ since it will not be relevant to the questions asked below)

$$L\, y'(t) \;+\; R\, y(t) \;+\; \frac{1}{C}\int_0^t y(\tau)\, d\tau \;=\; x(t).$$



   (a) Determine the transfer function, $H(s) \;=\; \dfrac{Y(s)}{X(s)}$, of the system.

   (b) Show that the system is stable for all (positive) values of the system parameters R, L, and C.

   (c) Determine the system frequency response $H(j\omega)$.

   (d) Referring to the frequency response, explain why the current will have no *dc* component provided that the capacitance is non-zero (and irrespective of whether the input voltage has a *dc* component or not).

   (e) Assume that the input voltage is a periodic function with period $T = \dfrac{1}{50}\,(s)$, that is $f_0 = \dfrac{\omega_0}{2\pi} = 50\,(Hz)$. For $0 \leq t < T$ it is given by

$$x(t) = \begin{cases} 10\,(V), & 0 \leq t < \dfrac{T}{2} \\ 0\,(V), & \dfrac{T}{2} \leq t < T \end{cases}$$

   Determine the exponential (complex) Fourier series for $x(t)$.

   (f) Set $R = 5\,\Omega$, $L = 40\,mH$, $C = 10\,\mu F$ and determine a Fourier-sum expression for the steady-state current, $y_s(t)$, in terms of the Fourier coefficients of the input voltage and the frequency response.

   (g) Plot a suitably truncated/approximate version of $y_s(t)$.
   How many terms are required for a (visually) convergent result ?

   (h) Plot $|H(j\omega)|$. Does this plot indicate a dominant frequency for the output current ?
   Is this consistent with the plot above ?

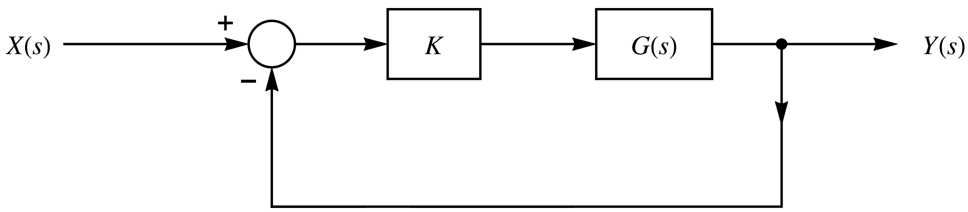
   (i) Set $R = 1000\,\Omega$, $L = 1\,mH$, $C = 1\,mF$ and plot the steady-state current again.

   (j) Plot $|H(j\omega)|$ and $\arg(H(j\omega))$.
   Explain why the shape of the current in this case closely mimics the shape of the input voltage.

2. The block diagram below represents a closed-loop system for position control (*e.g.* antenna tracking device), where the input, $x(t)$, represents a desired angular orientation, and the output, $y(t)$, is an observed actual orientation. The block G here represents a motor with given transfer function

$$G(s) = \frac{1}{s(s+4)}, \quad \text{and } K \text{ is a controllable, positive amplification.}$$

Since the input to the motor is proportional to the difference between desired and actual angle, the motor will then be in operation as long as the absolute value of this difference is larger than some small given value.



(a) Determine the transfer function of the system.

(b) Find and plot the unit step response of the system for values of $K = 5$, 40, and 85.

(c) Notice that with large values of K the system responds rapidly, which is, of course, a desirable feature. However, rapid response comes at the price of oscillations with an increasingly large initial overshoot. Assuming that we will tolerate a maximum overshoot of 30% - find the corresponding value of K.
Also, for this K-value, calculate the *settling time*, t_s, which we will define as a time after which the response stays within 99% of the desired value (here: $x(t) = \sigma(t) = 1$).

(d) Calculate the largest value of K for which the unit step response exhibits *no* oscillations and therefore approaches its desired value smoothly without overshooting. (For this particular value of the amplification the system is said to be *critically damped*). Also, for this K-value, determine the *rise time*, t_r, here defined as the time for the system to go from 10% to 95% of its steady-state value.

(e) Set $K = 40$. Find and plot the system response for inputs $x_1(t) = \sigma(t)\, t$ and $x_2(t) = \sigma(t) \cos t$.
Show that the steady-state values of $|x(t) - y(t)|$ are non-zero in these cases; that is, we will have to contend with small but persistent errors. (If the error is acceptable, the response for x_1 could be used in locking on an object moving with constant angular velocity with respect to the stationary tracking system).

3. The output of a LTID system/filter is given in terms of the input by

$$y[n] = \begin{cases} \max(x[n], x[n-1], \cdots x[0]), & n \leq N-1 \\ \max(x[n], x[n-1], \cdots x[n-(N-1)]), & n > N-1 \end{cases}$$

(a) Explain why the filter is causal.

(b) Write a Matlab function that takes a length-M ($> N$) vector x and a scalar N as input.
The output should be a length-M vector y.

(c) Test your code for the filter by filtering a $M = 50$ input given by $x[n] = \cos\left(\frac{n\pi}{5}\right) + \delta[n-30] - \delta[n-35]$.
Plot the result for $N = 4, 8, 12$.

(d) Determine the frequency response of the filter and plot the the amplitude response in the interval $0 \leq \Omega \leq \pi$.
Use the suggested values of N above. Is this a low- or high-pass filter ?